Computational challenges in assessment of marine resources

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Outline

- Mathematical ecology
  – Management of living resources
- Computation and statistics
  – Models
  – Numerical integration in high dimensions
  – Automatic differentiation
- Examples of ’assessment problems’
  – Harp seals

Institute of Marine Research

- Units
  - Bergen, Tromsø, Flødevigen, Austevoll, Matre
- 500 employees
  - 135 researches
- Center for marine resources
  - Fish (cod, herring, capelin, …)
  - Whales and seals

Management of living resources

- ‘Assessment’ = Estimation of
  - Current stock size
  - Biological parameters: mortality rates, birth rates, …
- Prediction of future stock size
  - Given today’s status
  - Under a given management regime (quota)
- Principles
  - Maximize harvest
  - Minimize risk of extinction
- Uncertainty
  - Important for determining risk

The complication

- Acoustic and trawl surveys do NOT give absolute estimates of stock size

$$I = q \cdot U$$

where

$$I = \text{survey index}$$

$$U = \text{stock size}$$

Population dynamics

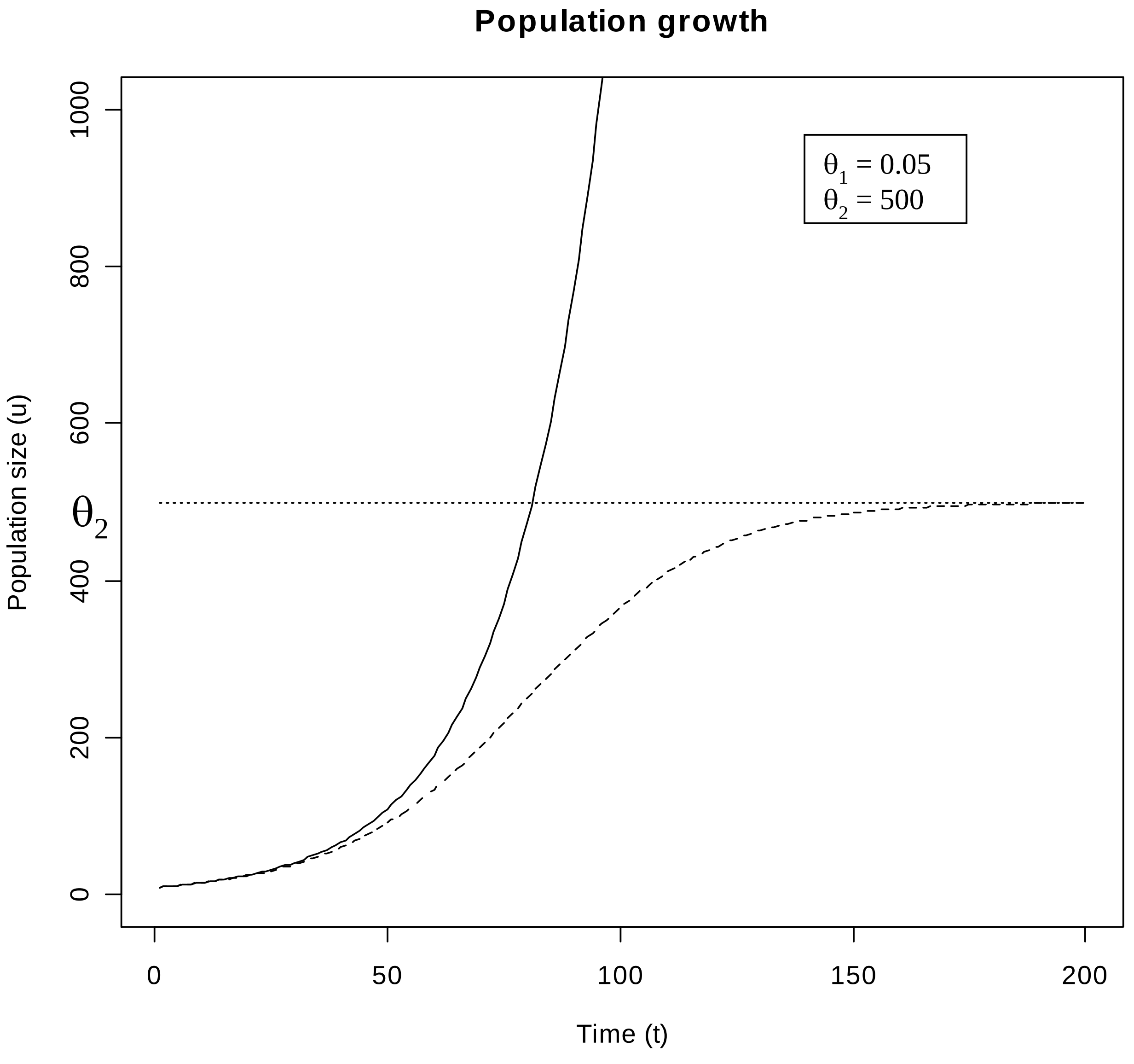
u_t = stock size at time t

- Exponential growth

$$u_t = u_{t-1} \cdot (1 + \theta_1)$$

- Density regulation

$$u_t = u_{t-1} \cdot [1 + \theta_1 (1 - u_{t-1} / \theta_2)]$$



Population growth
θ1 = 0.05
θ2 = 500
θ2
Population size (u)
Time (t)
0
200
400
600
800
1000
0
50
100
150
200

Population dynamics

- Exponential growth

$$u_t = u_{t-1} \cdot (1 + \theta_1)$$

- Density regulation

$$u_t = u_{t-1} \cdot [1 + \theta_1(1 - u_{t-1} / \theta_2)]$$

- Harvest

$$u_t = (u_{t-1} - c_{t-1}) \cdot [1 + \theta_1(1 - u_{t-1} / \theta_2)]$$

- Stochastic growth rate

$$u_t = (u_{t-1} - c_{t-1}) \cdot [1 + \theta_1(1 - u_{t-1} / \theta_2) + N(0, \theta_3)_t\ ]$$

Additonal complexity

- Age structure
- Spatial structure
- Interactions with other species
- Effect of environment

Models in statistics

- Parameters
  - θ — structural parameter, $\dim(\theta) \approx 20$
  - u — state parameter, $\dim(u) \approx 1000$
- Data
  - y — ‘measurements’
- Probability distributions
  - $p(y \mid u, \theta)$ — probability of data given u and θ
  - $p(u \mid \theta)$ — probability of u given θ
  - $p(\theta)$ — ’prior’ on θ

State-space model

- State equation $p(u \mid \theta)$

$$u_t = (u_{t-1} - c_{t-1}) \cdot [1+\theta_1(1- u_{t-1}/ \theta_2) + N(0, \theta_3)_t]$$

- Observation equation $p(y \mid u, \theta)$

$$y_t = \theta_4 \cdot u_t + N(0, \theta_5)_t$$

Models in statistics

- Joint probability distribution

$$p(y,u,\theta) = p(y|u,\theta) \cdot p(u|\theta) \cdot p(\theta)$$

- Marginal distribution

$$p(y,\theta) = \int p(y,u,\theta)\, du$$

Statistical computation

- Major challenge (today)

$$p(y,\theta) = \int p(y,u,\theta)\, du$$

- Maximum likelihood estimation θ

$$\underline{\theta} = \mathrm{argmax}_{\theta}\, p(y,\theta)$$

Algorithms

- Simple Monte Carlo integration
- Importance sampling
- Markov Chain Monte Carlo
  - Metropolis-Hastings algorithm
  - Gibbs sampler
- EM-algorithm
- Kalman filter (nonlinear)

Laplace approximation

$$p(y,\theta) = \int p(y,u,\theta)\, du$$

$$L(\boldsymbol{\theta}) = |\det(\mathbf{H})|^{-1/2} p(\mathbf{y}, \hat{\mathbf{u}}, \boldsymbol{\theta})$$

$$\hat{\mathbf{u}}(\theta) = \arg\max_{\mathbf{u}} p(\mathbf{y}, \mathbf{u}, \boldsymbol{\theta}),$$

$$\mathbf{H}(\theta) = \frac{\partial^2}{\partial \mathbf{u}^2} \log[p(\mathbf{y}, \mathbf{u}, \boldsymbol{\theta})]\,|_{\mathbf{u}=\hat{\mathbf{u}}_{\theta}}$$

Pierre-Simon Laplace (1749-1827)

- Bayes formula

$$p(u|y) = p(y|u) \cdot p(u) \,/\, p(y)$$

where

$$p(y) = \int p(y|u) \cdot p(u)\, du$$

- Replaces integration with maximization
- Exact for the Gaussian case

Maximum likelihood estimation

- Estimate θ by maximizing $L(\theta)$
- Gradient $\nabla_\theta L$ by automatic differentiation

AD (Automatic Differentiation)

- Given that we have a code for L
- AD gives you $\nabla_\theta L$
  - at machine precision
  - 'no' extra programming
  - efficiently (reverse mode AD)

$$\mathrm{cost}(\nabla_\theta L) \approx 4 \cdot \mathrm{cost}(L)$$

- AD people
  - Bischof, Griewank, Steihaug, ….
- Software: ADIFOR, ADOL-C, ADMB …

Laplace approximation again

$$L(\boldsymbol{\theta}) = |\det(\mathbf{H})|^{-1/2} p(\mathbf{y}, \hat{\mathbf{u}}, \boldsymbol{\theta})$$

where

$$\hat{\mathbf{u}}(\theta) = \arg\max_{\mathbf{u}} p(\mathbf{y}, \mathbf{u}, \boldsymbol{\theta}),$$

$$\mathbf{H}(\theta) = \frac{\partial^2}{\partial \mathbf{u}^2} \log[p(\mathbf{y}, \mathbf{u}, \boldsymbol{\theta})] |_{\mathbf{u}=\hat{\mathbf{u}}_{\boldsymbol{\theta}}}$$

Research questions

- AD versus analytical formulae
  - Which is the most efficient?
- Higher order derivatives (3th order)
  - Less efficient than the gradient
- Sparseness structure

Constrained optimization formulation

Maximize jointly w.r.t. u and θ

$$L(\boldsymbol{\theta}, \mathbf{u}) = \left|\det\left[\mathbf{H}(\mathbf{u}, \boldsymbol{\theta})\right]\right|^{-1/2} p(\mathbf{y}, \mathbf{u}, \boldsymbol{\theta})$$

under the constraint

$$\frac{\partial}{\partial \mathbf{u}} \log\left[p(\mathbf{y}, \mathbf{u}, \boldsymbol{\theta})\right] = 0$$

BeMatA project

- NFR- funded (2002-2005)
- Collaboration between:
  - Inst. of Marine Research
  - Inst. of informatics, UiB
  - University of Oslo
  - Norwegian Computing Center, Oslo
- Joins expertise in
  - Statistical modeling
  - Computer science / Optimization

Fisheries assessments

- Age determination of fishes invented around 1900
  - like counting growth layers in a tree
- ’Fundamental law’

$$u_{t,a} = (u_{t-1,a-1} - c_{t-1,a-1})\cdot(1 - \theta)$$

  - t is an index of year
  - a is an index of year
  - θ is mortality rate

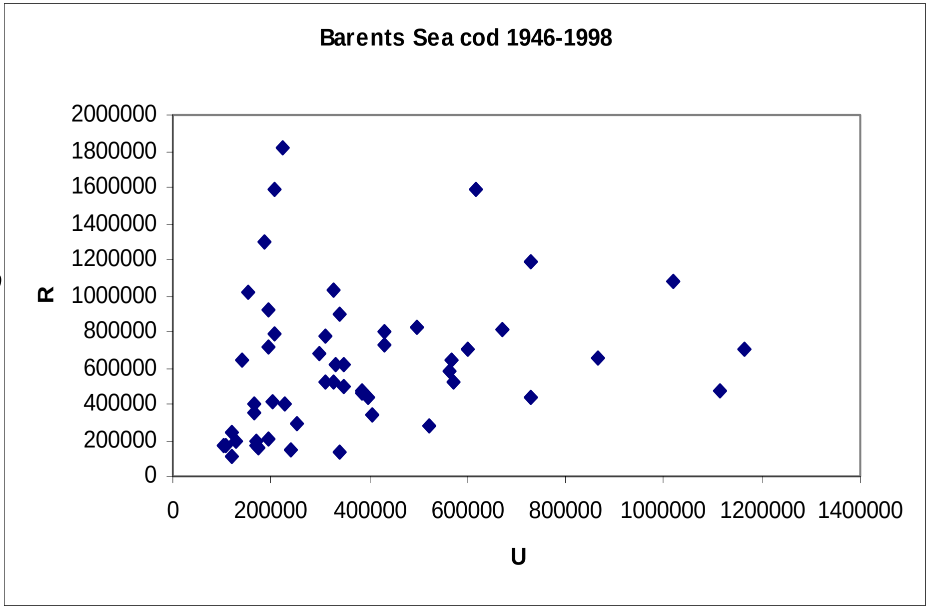
Recruitment mechanism

- Number of spawners

$$U_t = \Sigma_a \, u_{t,a}$$

- Number of fish 'born'

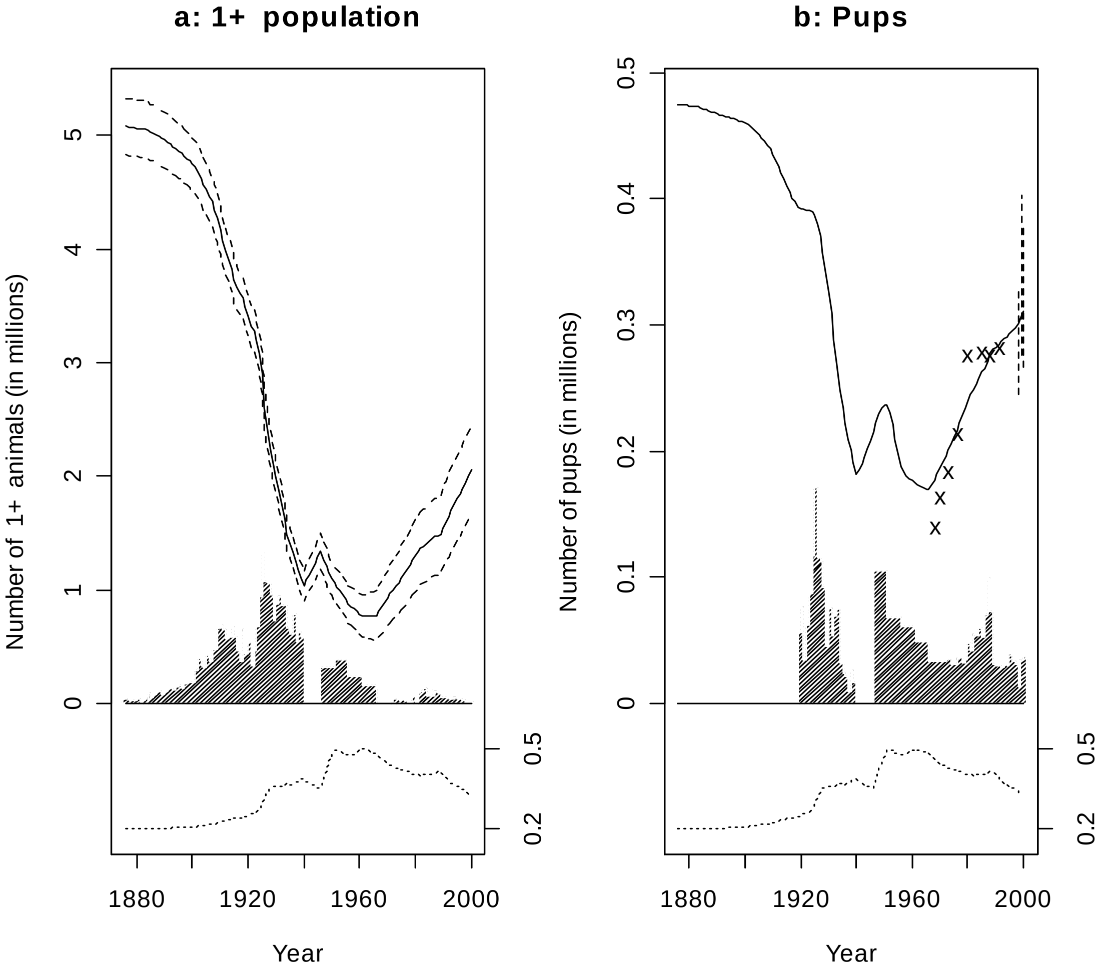
$$u_{t,0} \; = \; R(\, U_t, \, \theta \,)$$



- Strong relationship: seals
- Weak relationship: herring

Case study: Harp seals

- Barents Sea stock
  - Currently 2 million individuals
- Data
  - Historical catch records
  - Only pups can be counted
  - Age samples on whelping grounds
- Questions
  - Pre-exploitation stock size
  - Lowest level attained by the stock


a: 1+ population
b: Pups
Number of 1+ animals (in millions)
Number of pups (in millions)
0
1
2
3
4
5
0.5
0.2
0
0.1
0.2
0.3
0.4
0.5
1880
1920
1960
2000
Year

Annotated references

BeMatA project

Title: Latent variable models handled with optimization aided by automatic differentiation; application to marine resource

http://bemata.imr.no/

State-space models in fisheries

Gudmundsson, G. (1994). Time-series analysis of catch-at-age observations, *Applied Statistics*, **43**, p. 117-126.

AD

Griewank, A. (2000). Evaluating Derivatives: Principles and Techniques of Algorithmic Differentiation, *SIAM*, *Philadelphia*.

AD and statistics

Skaug, H.J. (2002). Automatic differentiation to facilitate maximum likelihood estimation in nonlinear random effects models. *Journal of Computational and Graphical Statistics*. 11 p. 458-470.

Laplace approximation in statistics

Tierney, L. and Kadane, J.B. (1986). Accurate approximations for posterior moments and marginal distributions. *Journal of the American Statistical Association* 81 p. 82-86